DYNAMIC SUBSTRUCTURING WITH MIXED BOUNDARY CONDITIONS TO COPE WITH COMPLEX STRUCTURAL ASSEMBLIES

R. Helfrich [(1)], J. Marchesini [(2)]

[(1)] INTES GmbH, Schulze-Delitzsch-Str.16, 70565 Stuttgart, Germany, Email: reinhard.helfrich@intes.de
[(2)] INTES France, 40 rue Sadi Carnot, 78120 Rambouillet, France, Email: jacques.marchesini@intes.fr

ABSTRACT

Dynamic substructuring is widely used in structural dynamics, where the complexity of large models prevent a direct modal solution of the full model in FE (Finite Element) analysis. Since the seventies, a standard method has been established using the Craig-Bampton method, which efficiently reduces parts of an assembly to a predefined number of internal modes and coupling DOF (degrees of freedom) at a fixed boundary. The main advantage of the method is its ease of use, which made it so popular. With increasing complexity of assembled models and raising frequency ranges, one important drawback of the Craig-Bampton method became more painful, i.e. the fixed boundary. Although methods exist to reduce part models with free boundaries they are hardly applied, because they are not easy to use.

1. USE OF DYNAMIC SUBSTRUCTURING

The use of dynamic substructuring facilitates the manipulation of large and complex structures by splitting the finite element model into several substructures. The condensation of each substructure allows having a reduced assembled model and decreases significantly the cost of calculation for dynamic analysis.

This approach is widely used in the frame of dynamic launcher applications. For coupled dynamic analysis of launch vehicle with payloads, the condensation method gives a high reduction of the launcher model size and allows the calculation of a huge number of variants in an affordable time (typically several hundreds of payload configurations), since the main computational effort is spent once during the condensation process.

Some additional applications with dynamic substructuring are the local analysis of main components like stages where the surrounding environment is a reduced model of the launcher in order to provide realistic boundary conditions at the component level.

Other use of dynamic reduction is for the MBD (Multi-Body Dynamics) analysis, where the focus lies on the smallest number of modes with highest accuracy of the modal basis.

2. USUAL CONDENSATION METHODS

There are numerous methods from the literature for dynamic reduction of substructures. As a first step, condensation methods need to partition substructures.

The degrees of freedom (DOF) of a substructure split into two parts as in Fig. 1:

- The interface DOF, or external DOF, or **C**oupled DOF, which link to the assembled model and could connect with surrounding structures.
- The internal DOF or **L**ocal DOF, which remain internal to the substructure and cannot connect with others structures.

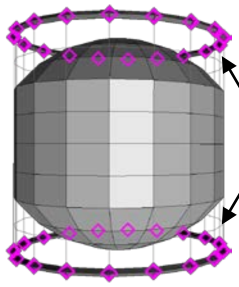

Figure 1. The substructure partitioning

Then the partitioned equation of motion of the isolated substructure is as follows (damping aspects not mentioned for simplification):

$$\begin{bmatrix} M_{CC} & M_{LC}^t \\ M_{LC} & M_{LL} \end{bmatrix} \begin{Bmatrix} \ddot{u}_C \\ \ddot{u}_L \end{Bmatrix} + \begin{bmatrix} K_{CC} & K_{LC}^t \\ K_{LC} & K_{LL} \end{bmatrix} \begin{Bmatrix} u_C \\ u_L \end{Bmatrix} = \begin{Bmatrix} F_C \\ F_L \end{Bmatrix} \qquad (1)$$

M, K, u, F are the mass matrix, the stiffness matrix, the displacement vector and the load vector, respectively, according to finite element theory.

The basic idea of the condensation method is to reduce the size of the substructure to the interface DOF by an appropriate transformation:

$$T^t M T \ddot{u}_C + T^t K T u_C = T^t F \qquad (2)$$

with T as transformation matrix.

2.1 Static reduction

The static reduction (or the condensation of Guyan [1]), is the base of the condensation methods. It is simply deduced from the static analysis:

$$\begin{bmatrix} K_{CC} & K_{LC}^t \\ K_{LC} & K_{LL} \end{bmatrix} \begin{Bmatrix} u_C \\ u_L \end{Bmatrix} = \begin{Bmatrix} F_C \\ F_L \end{Bmatrix} \tag{3}$$

Under certain conditions (i.e. the interface DOF must form a statically determinate support), the second row from this last equation expresses the local displacement as function of the interface displacements:

$$u_L = -K_{LL}^{-1} K_{LC} u_C + K_{LL}^{-1} F_L = -B_T u_C + K_{LL}^{-1} F_L \tag{4}$$

B_T is the restitution matrix. The first row from the equation (3) gives the formulation of the reduced system:

$$(K_{CC} - K_{LC}^t B_T) u_C = K_{red} u_C = F_C - B_T^t F_L \tag{5}$$

Then, the expression of the transformation matrix T is as follows:

$$\begin{Bmatrix} u_C \\ u_L \end{Bmatrix} = T u_C = \begin{bmatrix} I \\ -B_T \end{bmatrix} u_C \tag{6}$$

The static reduction gives an exact representation of the stiffness at the interface DOF, but the reduced stiffness matrix is almost fully populated. Thus, this method is efficient if the number of interface DOF remains small against the number of local DOF. The main computational effort in the reduction process is spent for the inversion of stiffness matrix for local DOF.

2.2 Dynamic condensation

For dynamic application, the static condensation gives inaccurate results with a high reduction scheme, where the transformation matrix produces an approximate reduced mass matrix.

In order to improve this situation, the dynamic reduction method of **Craig-Bampton** [2] extends the transformation matrix with normal eigenmodes of the substructure as follows:

$$u_L \approx -B_T u_C + X_L \eta \tag{7}$$

with X_L as normal eigenmode shapes from the substructure with **clamped** boundary conditions at the interface DOF and η the associated modal DOF. The term '$-B_T$' associated to the physical DOF is called the constraint modes.

This relation (7) depends on the number of normal modes used. Thus, the accuracy of this reduction is linked to modal truncation.

The expression of the transformation matrix T is now as follows:

$$\begin{Bmatrix} u_C \\ u_L \end{Bmatrix} = T u_C = \begin{bmatrix} I & \\ -B_T & X_L \end{bmatrix} \begin{Bmatrix} u_C \\ \eta \end{Bmatrix} \tag{8}$$

One interesting point of such method is the K-orthogonality of constraint modes with the normal clamped modes, which provides a consistent transformation matrix without any internal linear combination. Then, the transformation of the equation of motion from the expression (1) becomes as follows:

$$\begin{bmatrix} M_{red} & sym. \\ M_{\eta C} & \Lambda \end{bmatrix} \begin{Bmatrix} \ddot{u}_C \\ \ddot{\eta} \end{Bmatrix} + \begin{bmatrix} K_{red} & \\ & I \end{bmatrix} \begin{Bmatrix} u_C \\ \eta \end{Bmatrix} = \begin{Bmatrix} F_C - B_T^t F_L \\ X_L^t F_L \end{Bmatrix} \tag{10}$$

where M_{red} and K_{red} are the reduced mass and stiffness matrices on the interface DOF (identical from those given by Guyan's reduction), $M_{\eta C}$ the modal coupling mass matrix, and Λ the diagonal matrix of eigenvalues from clamped normal modes.

This method is simple to set up from the theoretical point of view and easy to use for the following reasons:

- This reduction does not lead to a rank deficiency problem.
- For static applications, this method simplifies itself into the Guyan's reduction, which provides the same stiffness at the interfaces DOF as that from the uncondensed model.
- For clamped vibrations at the interface DOF, the reduced model gives the same results as those from the uncondensed model.
- This method allows a high reduction level with few interface DOF.

From practical applications, such method gives stiffer results, and as the rule of thumb, the frequency limit for the normal clamped modes has to be two times higher than the analysed frequency of the assembled model.

Since this method uses a formulation with interface displacements (constraint modes), the connections between substructures remain natural and this reduction is in phase with the finite element method, where the substructure is seen as a 'super-element'.

The main advantage of the method is the ease of use, which made it so popular.

2.3 Current limitations and overview

Although the method of Craig-Bampton has interesting properties, there are some limitations in the treatment of complex models mainly due to the use of clamped internal modes in the following cases:

- For the behaviour of flexible substructures, the approximation by clamped internal modes could be

very poor and needs to raise the frequency limit of internal eigenfrequencies (three times or more against the analysed frequency).
- Such reduced model show a low convergence in dynamic analysis with free boundary conditions.
- The reduction of large substructure (like a launcher without the payloads) could need a very huge computational effort in order to reach the frequency limit.
- The reduction of large substructure with clamped boundary condition, like the condensation of a complete launcher at the payload interface, may lead to numerical inconstancies for dynamic response.
- This method gives a bad or no convergence with multiple interfaces (launcher with 2 payload interfaces).

There are numerous alternative dynamic reduction methods in the literature, for example [3]:
- Methods with clamped interfaces (Hurty, Craig-Bampton…)
- Methods with free interfaces (MacNeal, Rubin…)
- Methods with mixed boundary interfaces (Hintz…)
- Methods with loaded component modes (Benfield-Hruda)

Such methods carry some new advances but also have to tackle with different problems:
- Some interfaces of a reduced model cannot accept unsupported constraints, which prevent the use of free interface reduction.
- Some methods use the forces at interface DOF instead of the displacements, which need a specific treatment for the connection between substructures.
- Some of them have a limited range of applications and are considered as expert methods.

3. MIXED BOUNDARY CRAIG-BAMPTON METHOD

In order to improve the method of Craig-Bampton, the basic idea is to allow such method to handle any kind of internal eigenmodes. The use of a set of normal modes with the best adapted boundary condition will improve the convergence as well as the accuracy of the condensed models.

The expression of the transformation matrix T becomes as follows:

$$\begin{Bmatrix} u_C \\ u_L \end{Bmatrix} = T\, u_C = \begin{bmatrix} I & \\ -B_T & T_{L\eta} \end{bmatrix} \begin{Bmatrix} u_C \\ \eta \end{Bmatrix} \quad (6)$$

Where the modal restitution matrix $T_{L\eta}$ contains the information of internal modes calculated with any kind of boundary constraints. This matrix is built in a specific way to avoid internal linear combinations and to keep the property of K-orthogonality with the constraint modes, which ensures a consistent transformation and avoid the generation of a rank deficiency problem.

This formulation keeps the same architecture as the method of Craig-Bampton and conserves the properties linked to the constraint modes (like static reduction, connection via interface displacements).

This method was implemented in the PERMAS FE Software [4] under the name 'Mixed Boundary Craig-Bampton' method (MBCB) and it is seen as a natural extension of the method of Craig-Bampton with the same ease of use.

The next paragraphs will describe the different possibilities for the selection of normal eigenmodes.

3.1 Component modes reduction

The normal eigenmodes used for the reduction are built from the substructure level. In this case, the available boundary conditions are:
- Interfaces with fixed-boundary conditions,
- Interfaces with free-boundary conditions,
- Interfaces with mixed boundary conditions.

It is also possible to use a hybrid set of normal modes which could contain clamped modes and free modes.

In addition, the expansion of the normal mode basis with additional static mode shapes, allows the incorporation of any kind of specific modes, like those coming from the modal truncation augmentation method [5], in order to improve the accuracy of internal displacements. Such modes could be the static deflections from internal loads and the inertia relief attachment modes.

As it will be shown in chapter 4, this method converges exactly to the all-clamped boundary solutions with normal clamped eigenmodes (Craig-Bampton) but also to the all-free boundary solution with normal free eigenmodes.

3.2 Loaded component modes reduction

With the component mode reduction, the normal modes used for the reduction are built from the substructure level independent of the environment of this component. But in the assembled model, the substructure has some 'elastic' connections with surrounding parts.

With the loaded component modes reduction, the displacement shapes used for the reduction are the eigenmodes of the assembled model (substructure connected with other flexible components) projected to the substructure itself. An example is shown in Fig. 2.

But this projection involves that this set of displacement shapes is no longer linearly independent, since elastic eigenmodes from assembled model could have a quasi-rigid shape or could be collinear in the substructure region. Such singular or collinear displacement shapes are detected and removed before reduction.

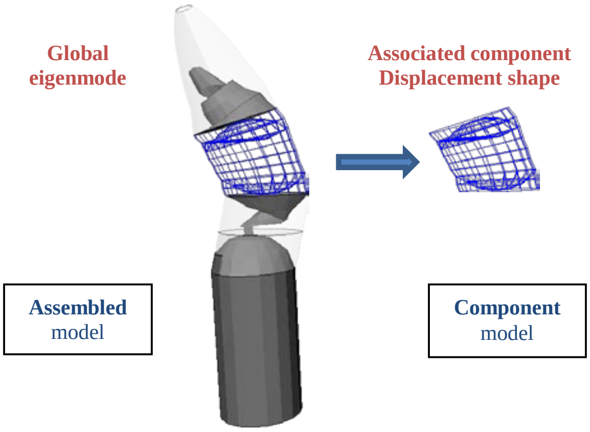

Figure 2. Loaded component mode

The component modes reduction is seen as a specific case of the loaded component modes reduction, which leads to a general formulation of the Mixed Boundary Craig-Bampton method.

4. NUMERICAL EXAMPLE: DRY TANK

This small example will show the component modes reduction. The structure is a cylindrical dry tank (height 2m, diameter 2m) with spherical end caps and short skirts ended by circular flanges. The finite element model, described in Fig. 3, has 18 nodes on its circumference and it is composed of linear shell elements and linear beam elements for the flanges.

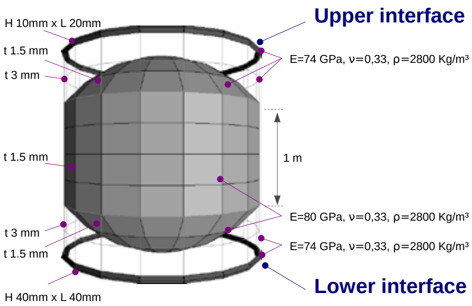

Figure 3. FE model of the dry tank

The eigenfrequencies are calculated up to 600 Hz. Table 1 shows the different boundary constraints to be analysed.

Constraints variants	Upper interface	Lower interface
FREE-FREE	Free	Free
CLAMPED-FREE	Free	Clamped
CLAMPED-CLAMPED	Clamped	Clamped

Table 1. Analysed constraints variants

Table 2 describes the different condensed models, which are reduced at lower and upper interfaces identically.

Models	Method	Internal modes	Frequency
CB	Craig & Bam.	165 clamped modes	1200 Hz
CBGEN	MBCB	81 free modes 6 inertia modes	650 Hz
CBGEN-V	MBCB	81 free modes 21 clamped modes 6 inertia modes	650 Hz 600 Hz

Table 2. Reduction schemes of dry tank

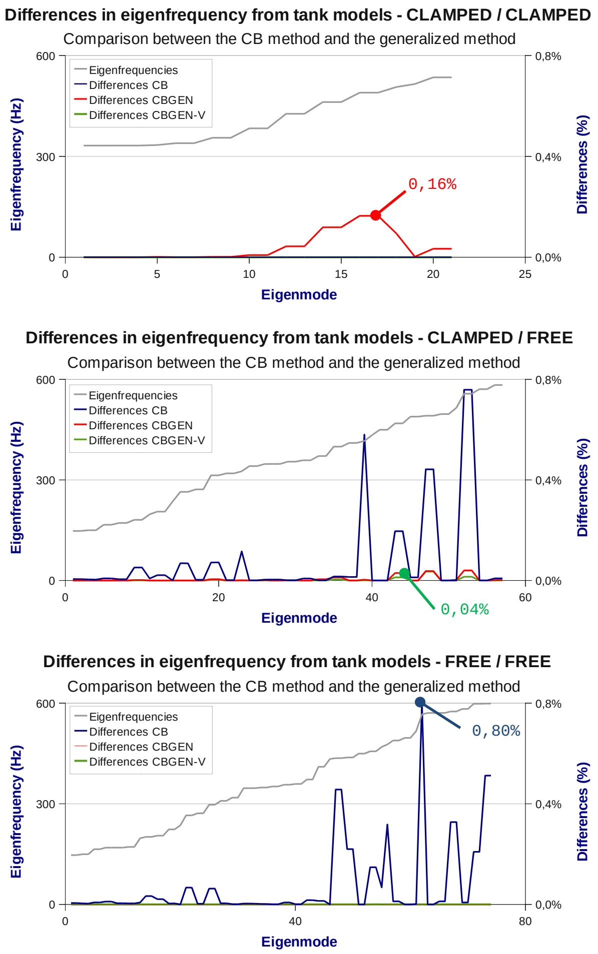

Figure 4. Comparison between reduction methods

The eigenfrequencies are compared to those given by the uncondensed model and the results are reported in the following curves (see Fig. 4). The worst cases are summarized in Table 3.

Models	Modal DOF	Worst case	Maximal deviation
CB	165	FREE-FREE	0,80%
CBGEN	81	CLAMPED-CLAMPED	0,16%
CBGEN-V	108	CLAMPED-FREE	0,04%

Table 3. Worst cases for condensed models

Like the Craig-Bampton reduction, which gives the exact solution for the CLAMPED-CLAMPED variant, the MBCB condensation with free normal eigenmodes gives also the exact solution for FREE-FREE variant.

The reduction with free eigenmodes (CBGEN) shows better results against the Craig-Bampton method with less modal DOF.

5. APPLICATION WITH LAUNCHER MODEL

5.1 Launcher A5GS – EAP stage separation

This model which represents the launcher A5GS, was developed by AIRBUS-DS, Les Mureaux, France. This application will show the loaded component mode reduction applied to a large component.

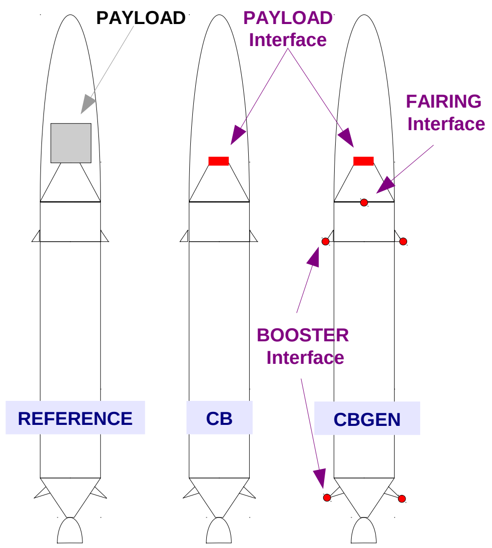

Figure 5. A5GS - Launcher reduction schemes

The goal is to compare the first 1250 coupled fluid-structure eigenfrequencies with free-boundary conditions from the following models:

- Reference launcher model with no condensation.
- Condensed launcher model with the method of Craig-Bampton and 2500 internal clamped eigenmodes (CB). The reduced model is built without the payload component and has only one interface with the payload. The internal eigenmodes are calculated with clamped boundary conditions at the payload interface.
- Condensed launcher model with the MBCB method and 1250 internal eigenmodes (CBGEN). The CBGEN model is built without the payload component and has interfaces with the payload, the fairing, and the connection points from EAP (Etage d'Acceleration à Poudre) boosters. The internal eigenmodes used for the condensation came from the reference launcher model (loaded component modes reduction).

The curve of eigenfrequencies with the index of mode from the reference launcher model is plotted in Fig. 6. In addition, the differences between the eigenfrequencies from the reference mode with those from the CB model and also with those from the CBGEN model are plotted in the same figure.

These results show a superior convergence from the MBCB method using the loaded component modes with less modal DOF and a maximum error less than 0.01%. For the method of Craig-Bampton, the maximum error is less than 3%.

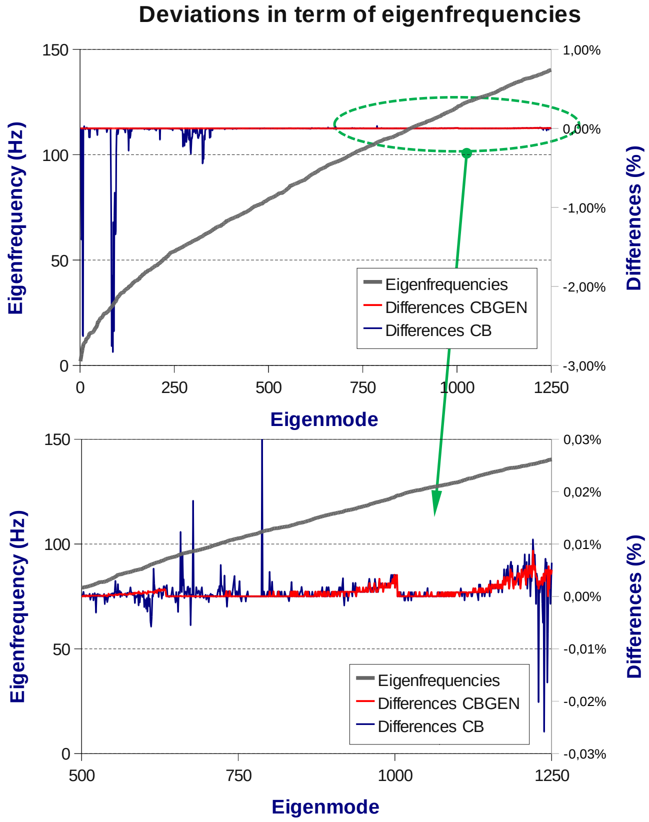

Figure 6. A5GS - Comparison between reduction methods

5.2 Launcher A5E/CA – Pressure oscillations from the first acoustic mode

This model which represents the launcher A5E/CA, was developed by AIRBUS-DS, Les Mureaux, France. This application will show the loaded component mode reduction applied to a multi-levelled assembly model.

The goal is to compare the first 800 coupled fluid-structure eigenfrequencies up to 80 Hz with free-boundary condition from the following models:

- The reference launcher model with no condensation.
- The condensed launcher model divided in several reduced substructures with the method of Craig-Bampton using clamped eigenmodes up to 200 Hz (CB).

- The condensed launcher model divided by the same reduced substructures but with the MBCB method using the 800 coupled eigenmodes from the reference launcher model (loaded component modes reduction) (CBGEN).

For the condensed models, the launcher is split into several substructures in the following way (see Fig.7):

- Booster components (EAP),
- Main liquid stage component (EPC),
- Engine component,
- Upper liquid stage component (ESC),
- Upper part with no condensation.

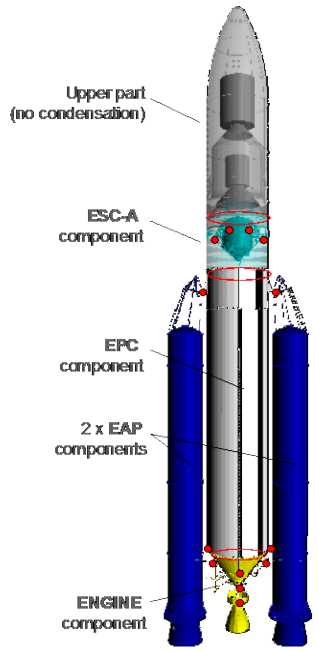

Figure 7. A5E/CA - Launcher reduction scheme

The internal eigenmodes used for the different reduced substructures are detailed in Table 4.

Reduced component	**Craig-Bampton method** Clamped modes ≤ 200 Hz	**MBCB method** Modes from the reference model ≤ 80 Hz
EAP (x2)	550	173 retained modes
ESC	500	272 retained modes
EPC	1 800	612 retained modes
ENGINE	250	204 retained modes
Sum of modal DOF	3 650	1 434

Table 4. AE/CA – Modal DOF

The curve of eigenfrequencies with the index of mode from the reference launcher model is plotted in Fig. 8. In addition, the differences between the eigenfrequencies from the reference mode with those from the CB model and also with those from the CBGEN model are plotted in the same figure.

These results show a superior convergence from the MBCB method using the loaded component modes with less modal DOF and a negligible maximum error. For the method of Craig-Bampton, the maximum error is about 0.2%.

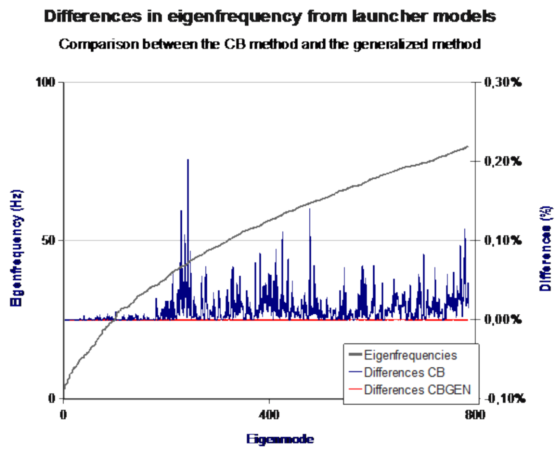

Figure 8. A5E/CA – Comparison between reduction methods

6. CONCLUSION

In order to overcome the limitations of the Craig-Bampton method in dynamic substructuring, an extension to free and mixed boundaries has been proposed. The Mixed Boundary Craig-Bampton (MBCB) method keeps the model reduction easy to use and improves the accuracy of the reduced models.

As shown by two launcher examples, the MBCB method is working for pure structural models and coupled fluid-structure models, where enclosed fluids are physically coupled with the surrounding structure.

Finally, it is obvious that the MBCB method will not only improve dynamic FE analysis but also MBD analysis using flexible part models.

7. REFERENCES

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