# **Robust Optimum in Structural Dynamics**

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#### Summary:

The first eigenfrequencies of a structure should be as far away as possible from a prescribed external excitation frequency - or band of excitation frequencies – to avoid resonance phenomena with high vibration and noise levels. On the other hand, weight reduction is one of the promising approaches to satisfy the new regulations of fuel economy and emissions. Apart from the trend to use lightweight materials an adaptation of wall thicknesses is used in the design process of car body structures. State-of-the-art optimization algorithms are capable to find solutions even in presence of opposing requirements. Moreover, the forced response amplitudes can be successfully reduced by optimization methods. However, design parameters such as wall thicknesses and material parameter are usually subjected to uncertainties due to manufacturing tolerances and measurement errors. Therefore, the main goal is to establish a robust design optimization that considers the scattering of the design parameters. The influence of the variability of selected parameters and their sensitivities on eigenfrequencies are investigated by a sampling procedure. Afterwards a sizing optimization procedure is presented. Finally, uncertain parameters are introduced and a reliability analysis of the initial model is conducted. All steps are illustrated by means of an accompanying body-in-white finite element model.

### Keywords:

Robust design, Eigenfrequency optimization, Reliability, Sampling

# 1 Introduction

The first eigenfrequencies of a structure should be as far away as possible from a prescribed external excitation frequency - or band of excitation frequencies – to avoid resonance phenomena with high vibration and noise levels. On the other hand, weight reduction is one of the promising approaches to satisfy the new regulations of fuel economy and emissions [7]. Apart from the trend to use lightweight materials an adaptation of wall thicknesses is often used in the design process of car body structures. State-of-the-art optimization algorithms are capable to find solutions even in presence of opposing requirements. Moreover, the forced response amplitudes can be successfully reduced by optimization methods. However, design parameters such as wall thicknesses and material parameter are usually subjected to uncertainties due to manufacturing tolerances and measurement errors. Therefore, the main goal is to establish a robust design optimization that considers the scattering of the design parameters. The influence of the variability of selected parameters and their sensitivities on eigenfrequencies are investigated by a sampling procedure. Afterwards a sizing optimization procedure is presented. Finally, uncertain parameters are introduced and a reliability analysis of the initial model is conducted. All steps are illustrated by means of an accompanying body-in-white example.

Azadi [1] investigated the improvement of a vehicle body structure under the constraint of noise, vibration and harshness (NVH) by using design of experiments (DOE) method. Costas [2] applied surrogate-based multi-objective optimization techniques to a crashworthiness problem. Sobieszczanski-Sobieski [10] carried out a size optimization problem of a full car body finite element model under constraints of NVH and crash. Usually, different finite element models have to be used for the crash and vibration analysis. A collection of finite element models for crashworthiness studies is available [13].

Fischer et.al. [5] and Gollwitzer et. al. [6] successfully applied a reliability analysis to a car body. Druesne [4] proposed a set of parametric numerical methods to predict the effect of uncertainties in the input parameters on the natural frequencies of structures.

Lyu [8] considered a multi-objective optimization problem of a three-dimensional finite element model of a vehicle body-in-white (BIW).

Yang [11] compared the mode shapes of a finite element analysis with experimentally measured mode shapes. Rashid [9] used a topology optimization to reduce the weight of a BIW model with additional constraints for two global mode shapes (torsion and bending).

Zuo [12] divided the design process of body-in-white (BIW) structure into two stages. The BIW structure is simplified by a thin-walled frame structure in the conceptual design stage, whereas a sophisticated finite element model is used in the detailed design stage.

# 2 Accompanying Example

The full scale finite element model is shown in Fig. 1.



Fig. 1: Body-in-white finite element model

No boundary conditions are applied here, i.e. 6 rigid body modes are theoretically present. However, due to numerical rounding errors not all rigid body modes might be detected. Therefore, the rigid body modes can be explicitely defined through \$RIGMODE definition by the user. Thus, spurious elastic modes are omitted. The quality of the rigid body modes is checked by the Rayleigh quotient. The model contains 86596 shell and beam elements (60281 QUAD4, 24937 TRIA3, 1378 BECOS). In total, the model consists of 401604 degrees of freedom. Different colors characterize the different parts of the structure. The couplings between the different parts is achieved by coincident nodes. Usually, various MPCs and spot welds can be used. Different procedures, such as SAMPLING, OPTIM and RELIABILITY ANALYSIS will be discussed in the subsequent sections with the prospect of gaining new scientific and technical knowledge of the NVH behavior of body-in-white finite element models. All

## 2.1 Sampling

dollar sign and capital letters in the subsequent sections.

A set of twenty-seven steel parts is selected for design elements. The wall thickness of each part is a design variable. Six different thickness values are used for the roof, whereas ten different thicknesses are applied to the undertray. The remaining parts are considered to be constant with respect to the thickness. It becomes evident, that the number of necessary loops is given by

computations are carried out in PERMAS. PERMAS specific commands are highlighted by a preceding

$$\prod_{i=1}^{27} n_{\nu,i} \quad , \tag{1}$$

where  $n_{v,i}$  denotes the number of discrete values specified for the i-th variable. Overall 60 data points are used within the sampling procedure. Thus a generalized eigenvalue problem

$$M(x) \Phi(x) = K(x) \Phi(x) \Lambda(x), \ x \in \mathbb{R}^{27}$$
(2)

for the parameter-dependent mass M(x) and stiffness matrix K(x) is repeatedly solved in one computational run. The material properties of all parts are kept constant in this study. However, the density  $\rho$  and Young's modulus *E* and even the shape of the car body can be considered as additional design parameters. Here, the influence of a thickness variation of the roof  $x_1$  and the undertray  $x_2$  on the first fundamental eigenfrequencies is depicted in Fig. 2.

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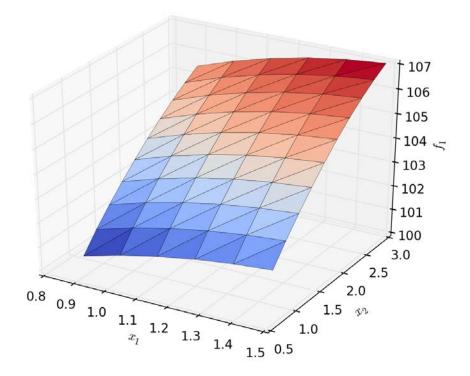


Fig. 2: Variability of the first eigenfrequency with respect to a thickness variation of the roof and undertray

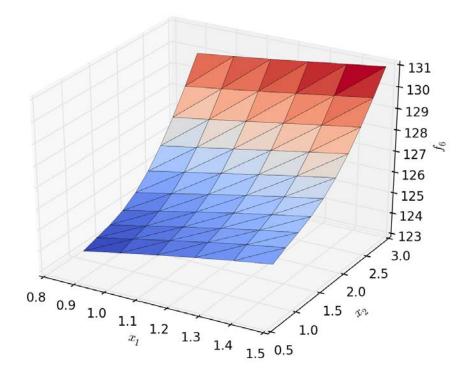
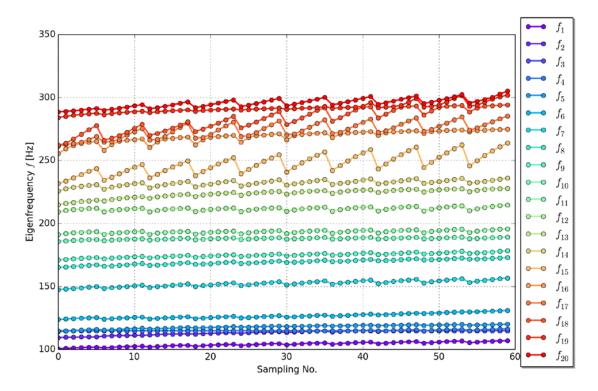


Fig. 3: Variability of the sixth eigenfrequency with respect to a thickness variation of the roof and undertray

A closer look at the first twenty eigenfrequencies in Fig. 4 reveals the so-called veering effect. Modecoupling and crossing of eigencurves can be observed.



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#### 2.2 Optimization

Mass minimization for the car body structure is achieved by the following optimization problem

$$\min_{\substack{x_{1} \leq x \leq x_{1}, \\ x_{1} \leq x \leq x_{2}, \\ x_{2} \leq x \leq x_{2}, \\ x_{3} \in \mathbb{C}}} \min_{\substack{x_{1} \leq x \leq x_{2}, \\ x_{2} \in \mathbb{C}}} \max_{x_{1} \in \mathbb{C}} \max_{x_{2} \in \mathbb{C}}$$

where f(x) is the objective function defined as the total mass of the car body. Additional constraints are represented by  $g_i(x)$ . Here a lower limit for the first eigenfrequency and a lower bound for the sixth eigenfrequency is defined. The design variables were allowed to vary between lower and upper bounds, denoted by  $x_l$  and  $x_u$ , respectively. A mode-tracking procedure is implemented in order to sort the real eigenvalues. Different types of mode shapes are inherently present in a car body model. These are local, global and combined modes. Local modes may be detected by a strain energy distribution plot. The elastic parts of the car body are illustrated by different colors. Each column represents an eigenfrequency of the system. Thus columns with a low number of different colors indicate local modes, whereas many different colors give a hint to global modes. This observation can be finally confirmed by looking at the corresponding mode shape.

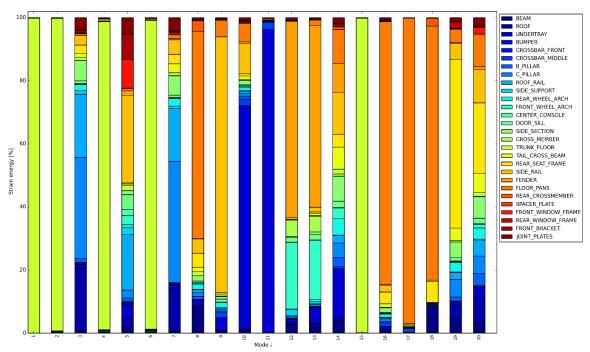


Fig. 5: Strain energy distribution for one configuration of wall thicknesses

The mode shape ID that is associated with the sixth eigenfrequency at the beginning of the optimization varies during the optimization. This effect is illustrated in Fig. 6. The last number in the corresponding legend denotes the current mode ID. The implemented mode-tracking procedure is able to detect mode switching by book-keeping. In addition messages of the form **Selected mode: 6 Current mode: 7** etc. are available in the cproject.log> file.

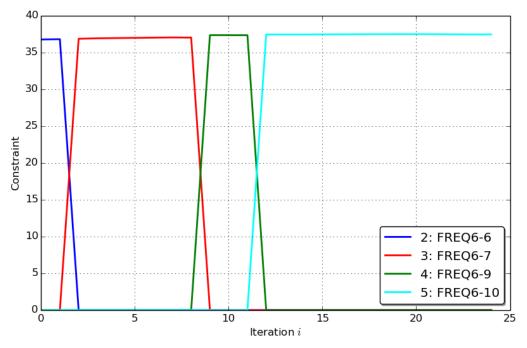


Fig. 6: Constraint history of the design constraint related to  $f_6$ 

#### 2.3 Reliability Analysis

Different numerical methods, such as FORM, SORM, RSM, adaptive and crude Monte-Carlo are directly available in PERMAS [14, 15]. Here we focus on FORM. Importance sampling may be used to improve the results of the first order method. The stochastic properties of the model are estimated. It was assumed, that the thicknesses of all sheets are uncertain due to tolerances and a uniform distribution was selected for each thickness. The support is defined by the two parameters, *a* and *b*, which are its minimum and maximum values. Since there are 27 independent sheet thicknesses, the stochastic model contains 27 basic variables. One or more failure functions can be defined to compute the probability of failure for the corresponding failure function. Each failure function g(Z, d) describes one mode of failure. The vector *Z* contains the state variables, which may be results of a finite element analysis or basic variables used as direct parameters, whereas vector *d* contains additional deterministic parameters. For g(Z, d) < 0 the structure fails, g(Z, d) = 0 is the limit state, and for g(Z, d) > 0 the structure is said to be in a safe state. The limit state function

$$g(Z) = f_1(Z) - \bar{f},$$

where  $f_1$  denotes the fundamental eigenfrequency of the car body and  $\bar{f}$  is a user-defined threshold value for the first eigenfrequency.

#### 3 Conclusions

Different tools to study the variability of eigenfrequencies due to uncertainties in wall thicknesses are available in PERMAS. The so-called sizing wizard in VisPER [16] can be used to complete the finite element model with respect to optimization relevant data input. Some additional entries for the SAMPLING and RELIABILITY ANALYSIS can be reused or easily added using a suitable text editor, e.g. an enhanced version of emacs [17].

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