Application of Optimization Methods in Rotor Dynamics

Reinhard Helfrich and Nils Wagner

INTES GmbH, Schulze-Delitzsch-Str. 16, D-70565 Stuttgart
Reinhard.helfrich@intes.de Nils.Wagner@intes.de
9th IFToMM International Conference on Rotor Dynamics,
September 22 - 25, 2014; Politecnico di Milano, Milan, Italy

Abstract. The purpose of this study is to investigate the optimal design of rotors with respect to different objective functions. The dynamic behavior of a rotating system is strongly influenced by various parameters such as mass and stiffness distribution, rigid disk inertial properties, bearing locations and coefficients. To a greater or lesser extent, all bearings are flexible and all bearings absorb energy. Moreover, load deflection relationships are often a function of shaft speed. An optimal design can be achieved by minimizing a selected target function subjected to specific constraints within a set of reasonable bounds for inner and outer diameters of the different stations, stiffness and damping coefficients of the support bearings and their positions. Optimal design of rotors has been reported by many authors in the literature. However, all of these optimization analyses considered beam elements instead of solid elements to the best knowledge of the authors. Nowadays three-dimensional models of rotors are common practice in the industry. Therefore solid models of rotating systems are considered in the current work. Stability criteria, critical speeds, weight, real and complex eigenfrequencies, and unbalance responses may be defined as a set of objective functions and constraints. In order to judge the stability, the equivalent damping ratio is evaluated. The determination of the damped critical speeds is accomplished using a Campbell diagram, a plot of damped natural whirl frequencies versus spin speeds, which is generated by determining the natural frequencies over a range of rotational speeds. A mode-tracking procedure is implemented in order to sort the complex eigenvalues. The complete finite element analysis including the complex design optimization problem of the rotor system at hand is conducted in PERMAS. For this purpose several optimization algorithms are available. Furthermore gradient-based and derivative free methods can be used. A combined shape and sizing optimization is used to illustrate the procedure by means of an example taken from the literature.

Keywords: Optimization, Rotor dynamics

1 Introduction

Finite element technique has become a popular tool in rotordynamic analysis. Dynamic studies of rotating machines are generally performed using, on the one hand, beam element models representing the position of the rotating shaft and, on the other hand, three-dimensional solid rotor-stator models. A specific advantage of solid models is the inclusion of stress stiffening, spin softening, and temperature effects in the rotor dynamics analysis. Nowadays CAD models of rotors becoming more and more detailed. The tedious and time-consuming task of building equivalent beam models is omitted by using solid models. Rotor lateral vibration is perpendicular to the axis of the rotor and is the largest vibration component in most high-speed machinery. Understanding and controlling this lateral vibration is important because excessive lateral vibrations leads to bearing wear and, ultimately, failure. In extreme cases, lateral vibration also can cause the rotating parts of a machine to come into contact with stationary parts, with potentially disastrous consequences. All FEM computations are carried out in PERMAS [25]. PERMAS specific commands are highlighted by a preceding dollar sign and capital letters in the subsequent sections.

2 Governing equations

Only linearized systems are considered here, i.e. only small variations of the rotational velocity is possible. Rotating systems may be processed in a stationary reference frame as well as a rotating reference frame. In the following, we will focus on an inertial reference frame. The additional matrices due to rotating parts must be taken into account and are requested by a so-called $ADDMATRIX data block within the $SYSTEM block. The complex eigenfrequencies of a rotor on fixed supports are determined. The structure is described with respect to a fixed reference frame, i.e. shaft and discs rotate with a constant rotational speed, whereas the bearings are supported and fixed to ground. All displacements, frequencies etc. refer to the fixed coordinate system. The first computation step is a static analysis for the basic model to determine the stress distribution under centrifugal loads. It is a prerequisite for the calculation of the geometric stiffness matrix K_g . The next step is the calculation of real eigenmodes $Y = [y_1, ..., y_r]$, including geometric and convective stiffness matrices:

$$MY = (K + K_b + K_g + K_c)\, Y\, \Lambda\,, \quad \Lambda = \begin{bmatrix} \lambda_1 & & \\ & \ldots & \\ & & \lambda_r \end{bmatrix}, \quad \lambda_i = \frac{1}{\omega_i^2}\,. \qquad (1)$$

The governing equations of motion that describes a rotor system in a stationary reference frame is given by

$$M\ddot{u} + (D + D_b(\Omega) + G)\,\dot{u} + (K + K_b(\Omega) + K_g + K_c)\,u = R(t)\,, \qquad (2)$$

where M denotes the mass matrix, D viscous damping matrix, $D_b(\Omega)$ (speed-dependent) bearing viscous damping matrix, G gyroscopic matrix, K structural stiffness matrix, K_c convective stiffness matrix, K_g geometric stiffness matrix, $K_b(\Omega)$

(speed-dependent) bearing stiffness matrix and $R(t)$ external forces. Including convective stiffness requires the use of a consistent mass matrix. The equations of motion (2) are transformed into modal space by means of

$$u = Y\,\eta\,. \tag{3}$$

Additional static mode shapes may be added to enrich the modal space. This is realized by $ADDMODES. If the free response of the system is considered $R(t)=0$. To analyze rotating models two different coordinate systems can be used in PERMAS, stationary and rotating. By using a stationary reference frame the model can have both rotating parts and stationary parts. However, the rotation parts have to be axisymmetric. A stationary reference frame is activated by

```
$ADDMATRIX
GEOSTIFF CONVSTIFF GYRO
```

The rotational speed is defined in the loading definition of a static pre-run by $INERTIA ROTATION. Additional matrices are built for that reference speed. The modelling of a rotating machine requires a skew-symmetric pseudo-damping matrix named gyroscopic matrix. The particular form of the matrix makes complex eigenmodes appear, forward modes having increasing frequencies and backward modes having decreasing frequencies. Identical damping specifications lead to different effects in fixed or co-rotating reference systems. In an inertial reference frame material damping is not suitable for rotating parts, whereas modal damping represents any kind of external damping. Discrete damping elements can be used for modelling damping in bearings. In order to get the relation between eigenfrequencies and rotational speed an automatic procedure called $MODAL ROTATING is available which directly generates all eigencurves. A mode tracking algorithm is implemented in order to sort the complex eigenvalues. It becomes evident that mode-tracking is an important feature. It was shown, that the backward mode vector is orthogonal to the unbalance vector and, as such, energy cannot be fed into the backward whirl. Therefore, critical speeds are restricted to forward whirl in case of symmetric rotors.

3 Example

The example is taken from the literature [22].

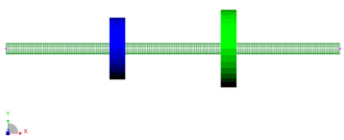

Fig. 1. The layout of the two-disk, two-bearing rotor

The finite element model consists of (18792 hexahedral, 5508 pentahedron and two spring-damper elements) Fig. (1). The elastic discs are incompatible meshed and connected to the elastic shaft by $MPC ISURFACE definitions. The bearings are idealized by spring-damper elements which are connected to the shaft by $MPC WLSCON elements. Again the coupling to the shaft is realized by $MPC ISURFACE. Therefore no mesh distortion is present, when the positions of the discs and bearings are updated during the optimization. Sizing parameters are the stiffness x_9 and damping x_{10} coefficients of the bearings, whereas shape variables are the positions of the two bearings x_1, x_8 and the elastic discs x_3, x_6. Moreover the elastic discs are free to change their thickness x_4, x_7 and outer diameter x_2, x_5, respectively. Overall ten different continuous design variables are available. The Campbell diagram for the initial rotor is depicted in Fig. (2)

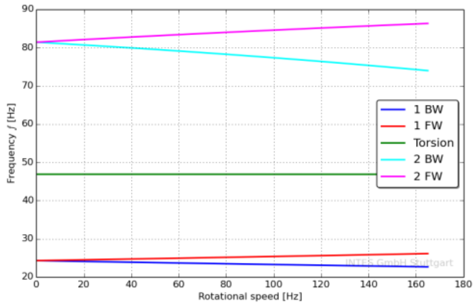

Fig. 2. Campbell diagram for the initial configuration

4 Optimization

Optimal design of rotor systems has been reported by many authors. A part of these studies focused the optimization process on the geometrical characteristics of the rotor [23], considering the bearing supports as idealized springs with non-speed dependent stiffness coefficients. Previous works in rotordynamic optimization applications with eigenfrequency constraints are done by [4], [13], [17]. The strain energy, either at a certain speed of operation or over a range of speeds is used as the objective function in [12]. Optimum weight design is discussed in [11], [15], [18]. The optimal placement of critical speeds is studied in [16]. Multi-objective optimization is analyzed by [3], [6]. Different optimization methods, such as genetic algorithms [2], [23], evolutionary [1], gradient-based algorithms [15] and LMI-based optimization [7] are used. Unlike methods based on sensitivity analysis and gradient descent, evolutionary algorithms are particularly effective at finding solutions with global optimality. In this study an optimization algorithm based on [14] is used. The box constrained-problem is of the form

$$\max\ f_1(x),\ x_l \le x \le x_u,\ x \in R^{10},\ s.t.\ g(x) \le 0, \tag{4}$$

where x_l is a vector of lower bounds, and x_u is a vector of upper bounds of the corresponding design variable. Inequality constraints are denoted by $g(x)$. A list of different design constraints is directly available in a rotordynamic analysis, e.g. $DCONSTRAINT ACCE, CFREQ, CAMPBELL, COMPLIANCE, DISP, ELSTRESS, NPSTRESS, FREQ, REAC, VELO, WEIGHT. Moreover, user defined constraints can be arbitrarily defined by using $DCFUNCTION. The objective function of an optimization may be the weight or any other specified constraint from the previous list. For frequency response optimization magnitudes, phases, real and imaginary parts of displacements, velocities and accelerations are available for constraints or objective definition. The design constraint CAMPBELL can be used to optimize the equivalent damping ratio for certain mode shapes for a given range of rotational speeds. In this work, we focus on real eigenfrequency optimization, i.e. the fundamental eigenfrequency f_1 is maximized. It is quite evident, that the critical speed can be increased if the fundamental eigenfrequency can be raised Fig. (2) and Fig. (5). The objective function $f_1(x)$ and some higher eigenfrequencies are illustrated in Fig. (3). Due to the symmetry of the rotor bending modes appear pairwise. The side constraints, initial values $x_0^T = [x_1,\ldots,x_{10}]$ and optimized values x_{opt} are specified in Table 1

Table 1. Nominal and optimal design

Design variable	Initial value	Optimized value	Lower bound	Upper bound
x_1 [mm]	0	50.0	0	50
x_2 [mm]	0	-30.0	-30.0	60
x_3 [mm]	0	70.0	-70.0	70
x_4 [mm]	0	-30.0	-30.0	30
x_5 [mm]	0	-3.0	-30.0	30
x_6 [mm]	0	-70.0	-70.0	70
x_7 [mm]	0	-30.0	-30.0	30
x_8 [mm]	0	50.0	0	50
x_9 [N/mm]	2.e5	2.689e5	1.8e5	3.0e5
x_{10} [Ns/mm]	10.0	10.22	9.0	11.0

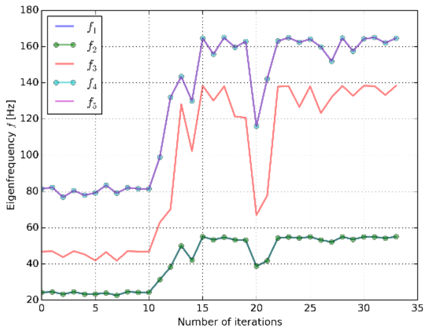

Fig. 3. The first five eigenfrequencies of the undamped system

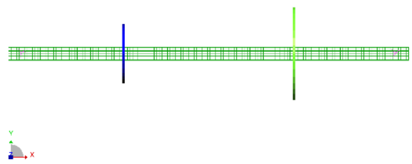

Fig. 4. Optimized shape of the rotor

4.1 Design variable history

In order to maximize the first eigenfrequency the mass should be minimized, whereas the stiffness should be maximized. This requirement is in relationship with the Rayleigh quotient. The Campbell diagram for the optimal solution is depicted in Fig. (5).

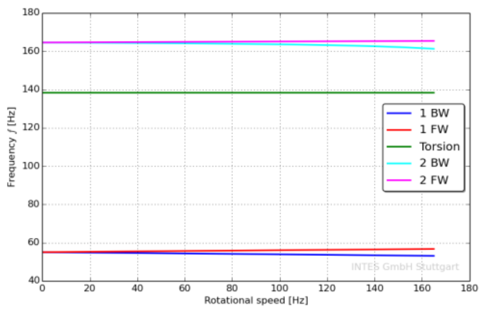

Fig. 5. Campbell diagram for the optimized configuration

The bearings move towards the upper and lower limits, respectively Fig. (6).

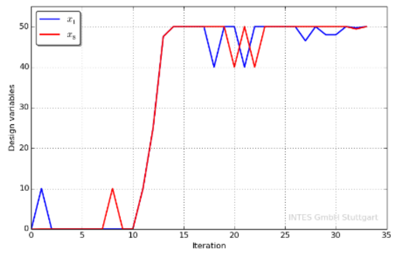

Fig. 6. Position of the bearings x_1, x_8 [mm]

The elastic discs tends to move towards the left end of the shaft within the given intervals Fig. (7).

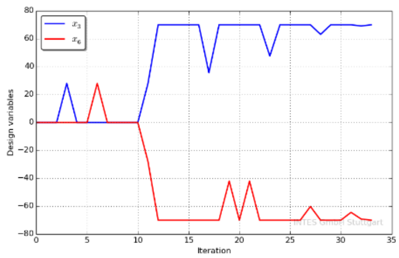

Fig. 7. Position of the discs x_3, x_6 [mm]

In addition, the elastic discs shrink during the optimization with respect to diameter Fig. (8) and thickness Fig. (9)

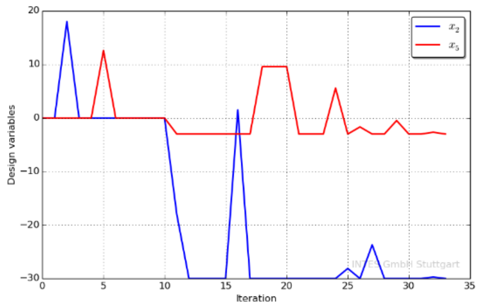

Fig. 8. Radial expansion of the discs x_2, x_5 [mm]

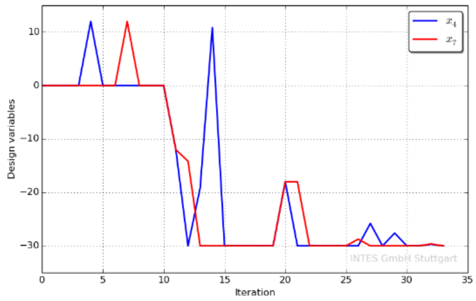

Fig. 9. Axial expansion of the discs x_4, x_7 [mm]

The mass of the considered rotor is drastically reduced since the elastic discs shrink in both, radial and axial, direction during the optimization Fig. (10). Additional dynamic constraints, e.g. response amplitudes over a certain frequency band, can be introduced to limit the displacements. Moreover design constraints like from limited stresses will reduce the drastic loss of weight. Thus an industrially feasible design can be achieved. However, this is not taken into account here.

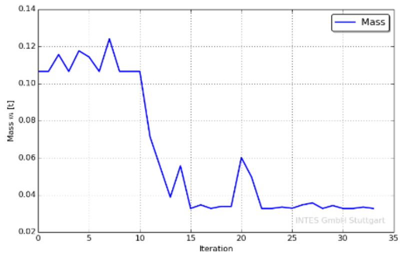

Fig. 10. Evolution of mass m [t]

5 Concluding Remarks and Outlook

This article presented an optimization methodology for viscoelastically supported rotor systems. The goal of the optimization problem was to find the values of a set of design variables (e.g. stiffness of the bearing thickness and outer diameters of the discs, positions of discs and bearings) for which the first natural frequency of the system is maximized. The optimized rotor reveals less weight and higher critical speeds. Flexible rotors are characterized by inherent uncertainties affecting the parameters that influence the dynamic response of the system. Therefore the handling of variability in rotor dynamics is a natural and necessary extension of the modeling capability [9], [20]. This will be investigated by a robust design analysis in the near future, where the reliability is used as a design constraint. In a first step, the influence of various design variables on the dynamic behavior of rotors will be investigated by a so-called sampling procedure. This approach offers a seamless scanning of the design space in one computational run.

6 References

1. Angantyr, A. Aidanpää, J.-O. : Optimization of a rotor-bearing system with an evolutionary algorithm. The 10th International Symposium on Transport Phenomena and Dynamics of Rotating Machinery, Honolulu, Hawaii, March 07-11 2004
2. Angantyr, A, Aidanpää, J.-O. : A Pareto-based genetic algorithm search approach to handle damped natural frequency constraints in turbo generator rotor system design. Journal of Engineering for Gas Turbines and Power 126, 619—625 (2004)
3. Angantyr, A, Aidanpää, J.-O. : Constrained optimization of gas turbine tilting pad bearing designs. Journal of Engineering for Gas Turbines and Power 128, 873—878 (2006)
4. Chen, T. Y., Wang B. P.: Optimum design of rotor-bearing systems with eigenvalue constraints. Journal of Engineering for Gas Turbines and Power 11, 256—260 (1993)
5. Choi, B. G., Yang, B. S.: Optimum shape design of rotor shafts using genetic algorithm. Journal of Vibration and Control, 6, 207—222 (2000)
6. Choi, B. G.: Multi-objective optimum design of rotor-bearing systems with dynamic constraints using immune-genetic algorithm. Journal of Engineering for Gas Turbines and Power 123, 78—81 (2001)
7. Cole, M. O. T, Wongratamaphisan, T., Keogh, P. S.: On LMI-based optimization of vibration and stability in rotor system design. Journal of Engineering for Gas Turbines and Power 128, 677—684 (2006)
8. Huang, S.-C., Lin, C. A.: Sensitivity analysis and optimization of undamped rotor-critical speeds to supports stiffness. Journal of Vibrations and Acoustics 126, 296--301 (2002)
9. Koroishi, E.H., Cavalini, A.A., de Lima, A. M. G., Steffen, V.: Stochastic modeling of flexible rotors. J. of the Braz. Soc. of Mech. Sci. & Eng. 34, 574--583 (2012)
10. Lee, D. S., Choi, D. H.: Reduced weight design of a flexible rotor with ball bearing stiffness characteristics varying with rotational speed and load. Journal of Vibration and Acoustics 122, 203—208, (2000)
11. Lin, Y.H., Lin, S.C.: Optimal weight design of rotor systems with oil-film bearings subjected to frequency constraints. Finite Elements in Analysis and Design 37, 777--798 (2001)
12. Lin, Y., Cheng, L., Huang, T.P.: Optimal design of complex flexible rotor-support systems using minimum strain energy under multi-constraint conditions. Journal of Sound and Vibration 215, 1123—1134 (1998)
13. Lopez, R.H., Ritto, T.G., Sampaio, R., Souza de Cursi, J. E.: A new algorithm for the robust optimization of rotor-bearing systems. Eng. Opt. 46, 1123--1138 (2014)
14. Marazzi, M., Nocedal, J.: Wedge trust region methods for derivative free optimization. Mathematical Programming Series A 91, 289--305 (2002)
15. Pugachev, A.: Application of gradient-based optimization methods for a rotor system with static stress, natural frequency, and harmonic response constraints. Structural and Multidisciplinary Optimization 47, 951--962 (2013)
16. Rajan, M., Rajan, S. D., Nelson, H. D., Chen, W. J.: Optimal placement of critical speeds in rotor-bearing systems. Journal of Vibration Acoustics, Stress, and Reliability in Design 109, 152--157 (1987)

17. Ritto, T., Lopez, R.H., Sampaio, R., Souza de Cursi, J. E.: Optimization of a flexible rotor-bearing system considering uncertainties. Proceedings of PACAM XI, Foz do Iguacu, PR, Brazil (2010)
18. Shiau, T.N., Hwang, J. L.: Optimum weight design of a rotor bearing system with dynamic behavior constraints. Journal of Engineering for Gas Turbines and Power 112, 454--462 (1990)
19. Shiau, T.N., Chang, J.R.: Multi-objective optimization of rotor-bearing system with critical speed constraints. Journal of Engineering for Gas Turbines and Power 115, 246--255 (1993)
20. Stocki, R., Szolc, T., Tauzowski, P., Knabel, J.: Robust design optimization of the vibrating rotor shaft system subjected to selected dynamic constraints. Mechanical Systems and Signal Processing 29, 34--44 (2012)
21. Untaroiu, C. D., Untaroiu, A.: Constrained design optimization of rotor-tilting pad bearing systems. Journal of Engineering for Gas Turbines and Power 132, (2010)
22. Wagner, N., Helfrich R.: Useful tools in the design process of rotor-bearing systems. NAFEMS Nordic Conference, May 13-14 Gothenburg, Sweden (2014).
23. Wang, J. H., Shih, F. W.: Improve the stability of rotor subjected to fluid leakage by optimum diameters design. Journal of Vibration and Acoustics 112, 59—64 (1990)
24. Yang, B. S., Choi, S. P., Kim, Y. C.: Vibration reduction optimum design of a steam-turbine rotor-bearing system using a hybrid genetic algorithm. Struct. Multidisc. Optim. 30, 43--53 (2005)
25. PERMAS Product Description, http://www.intes.de/kategorie_unternehmen/publikationen