Useful Tools in the Design Process of Rotor-Bearing Systems

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1 Introduction

Due to demands for high-performance machines many types of modern rotating systems are designed to operate above one or more critical speeds. Already during the design stage of a rotor, it is essential to understand the effect of the bearing stiffness on the critical speeds. The critical speed map characterizes the evolution of the critical speeds of the rotor with respect to the bearings stiffness.
Thus critical speed maps help engineers to evaluate a machinery rotor system. The critical speed map shows where critical speeds will likely occur. Moreover it also reveals the likely effectiveness of the changing stiffness of bearings or bearing supports in changing a critical speed. In order to generate such a map an eigenvalue problem must be solved for different values of bearing stiffness. In this case, the supports are treated as isoelastic springs equal at each location. Then the support stiffness is varied over many decades, since critical speed maps generally use a log-log scale.
For that purpose a sampling procedure is established in PERMAS [5] to simplify everyday life of a rotordynamic engineer. The first computation step is a static analysis for the basic model to determine the stress distribution under centrifugal loads. It is a prerequisite for the calculation of the geometric stiffness matrix.
The next step is the calculation of real eigenmodes including geometric and convective stiffness matrices. These mode shapes are used to transform the equations of motion into modal space. Additional residual mode shapes may be added to enrich the modal space.
The final step is a complex eigenvalue analysis to compute the corresponding Campbell diagram. A mode tracking procedure is implemented in order to guarantee smooth curves. Predicted critical speeds are indicated by the intersection of the once-per-revolution synchronous line and the resonance frequencies as functions of rotor speed. These critical speeds are collected for preselected stiffness values to yield the critical speed map. An example from the literature is used to illustrate the effectiveness of the approach. All FEM computations are carried out in PERMAS. PERMAS specific commands are highlighted by a preceding dollar sign and capital letters in the subsequent sections.

1.1 Equations of Motion

Only linearized systems are considered here, i.e. only small variations of the rotational velocity is possible. Rotating systems may be processed in a stationary reference frame as well as a rotating reference frame.
In the following, we will focus on an inertial reference frame. The additional matrices due to rotating parts must be taken into account and are requested by a so-called $ADDMATRIX data block within the $SYSTEM block.
The complex eigenfrequencies of a rotor on fixed supports are determined. The structure is described with respect to a fixed reference frame, i.e. shaft and discs rotate with a constant rotational speed, whereas the bearings are supported and fixed to ground. All displacements, frequencies etc. refer to the fixed coordinate system.
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The next step is the calculation of real eigenmodes

$$M\,X = (K + K_g + K_c)\;X\;\Lambda\,, \quad \Lambda = \begin{bmatrix} \lambda_1 & & \\ & \dots & \\ & & \lambda_r \end{bmatrix} \tag{1}$$

Including geometric and convective stiffness matrices: The governing equations of motion that describes a rotor system in a stationary reference frame is given by

$$M\ddot{u} + (D + D_b(\Omega) + G)\,\dot{u} + (K + K_b(\Omega) + K_g + K_c)\;u = R(t)\,, \tag{2}$$

where M denotes the mass matrix, D viscous damping matrix, D_b speed-dependent bearing viscous damping matrix, G gyroscopic matrix, K_c convective stiffness matrix, K_g geometric stiffness matrix, $K_b(\Omega)$ speed-dependent bearing stiffness matrix. Material damping of the stator is replaced by an equivalent viscous damping in the time-domain. Including convective stiffness requires the use of a consistent mass matrix, which is the default formulation beginning with version 14. The equations of motion (2) are transformed into modal space by means of

$$u = X\;\eta\,. \tag{3}$$

Additional static mode shapes may be added to enrich the modal space. This is realized by $ADDMODES.

$$\tilde{M}\ddot{\eta} + (\tilde{D} + \tilde{D}_b(\Omega) + \tilde{G})\,\dot{\eta} + (\tilde{K} + \tilde{K}_b(\Omega) + \tilde{K}_g + \tilde{K}_c)\;\eta = \tilde{R}(t)\,. \tag{4}$$

By introducing $\xi = \dot{\eta}$ the second order form (4) is transformed into a state-space form:

$$\begin{bmatrix} \tilde{M} & O \\ O & I \end{bmatrix}\begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} + \begin{bmatrix} \tilde{D} + \tilde{D}_b + \tilde{G} & \tilde{K} + \tilde{K}_b + \tilde{K}_g + \tilde{K}_c \\ -I & O \end{bmatrix}\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} \tilde{R}(t) \\ 0 \end{bmatrix} \quad . \tag{5}$$

To analyze rotating models two different coordinate systems can be used in PERMAS, stationary and rotating. By using a stationary reference frame the model can have both rotating parts and stationary parts. However, the rotation parts have to be axisymmetric. Moreover different components can rotate with different rotational speeds. A stationary reference frame is activated by

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$ADDMATRIX
 GEOSTIFF CONVSTIFF GYRO
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The rotational speed is defined in the loading definition of a static pre-run by $INERTIA ROTATION-Additional matrices are built for that reference speed.
The modelling of a rotating machine requires a skew-symmetric pseudo-damping matrix named gyroscopic matrix. The particular form of the matrix makes complex eigenmodes appear, forward modes (FW) having increasing frequencies and backward modes (BW) having decreasing frequencies. Critical speeds were evaluated in the operating speed range.

2 Campbell Diagram

In order to get the relation between eigenfrequencies and rotational speed an automatic procedure called $MODAL ROTATING is available which directly generates all eigencurves. A mode tracking algorithm is implemented in order to sort the complex eigenvalues. The Campbell diagram for an example taken from the literature [2] is depicted in Fig. (1). Usually the natural frequency of the BW mode decreases as the rotation speed increases. Correspondingly, the natural frequency of the FW mode increases as the rotation speed increases. Putative candidates for critical speeds are given by the intersections between the eigencurves and the synchronous line. Usually, critical speeds are excited by unbalance forces that affect the rotor. If bearing stiffnesses are symmetrical in two radial directions, unbalance forces cannot excite the backward whirling modes [3, 4].
The general principles for finding torsional critical speeds are similar to those for lateral critical speeds. However, there are some specific differences. The torsional natural frequencies are independent of spin speed so they plot as horizontal lines on the Campbell diagram. However, due to numerical reasons there are some small fluctuations in the torsional natural frequencies with respect to spin

speed. Therefore the corresponding values are scaled by their mean values. Then the differences between consecutive elements of each torsional eigencurve are added. If the sum is below a threshold value, the corresponding eigencurve is associated with a speed-independent mode. That criterion is used to filter them out here.
The lateral eigencurves are approximated by cubic splines to compute the intersections between the corresponding eigencurve and the synchronous line. This is done by bisection.
The procedure is repeated for increasing values of the support stiffness by using the new SAMPLING method that is available since PERMAS version 15. All critical speeds are collected.
Finally a critical speed map [7, 8, 10] is available Fig. (2).

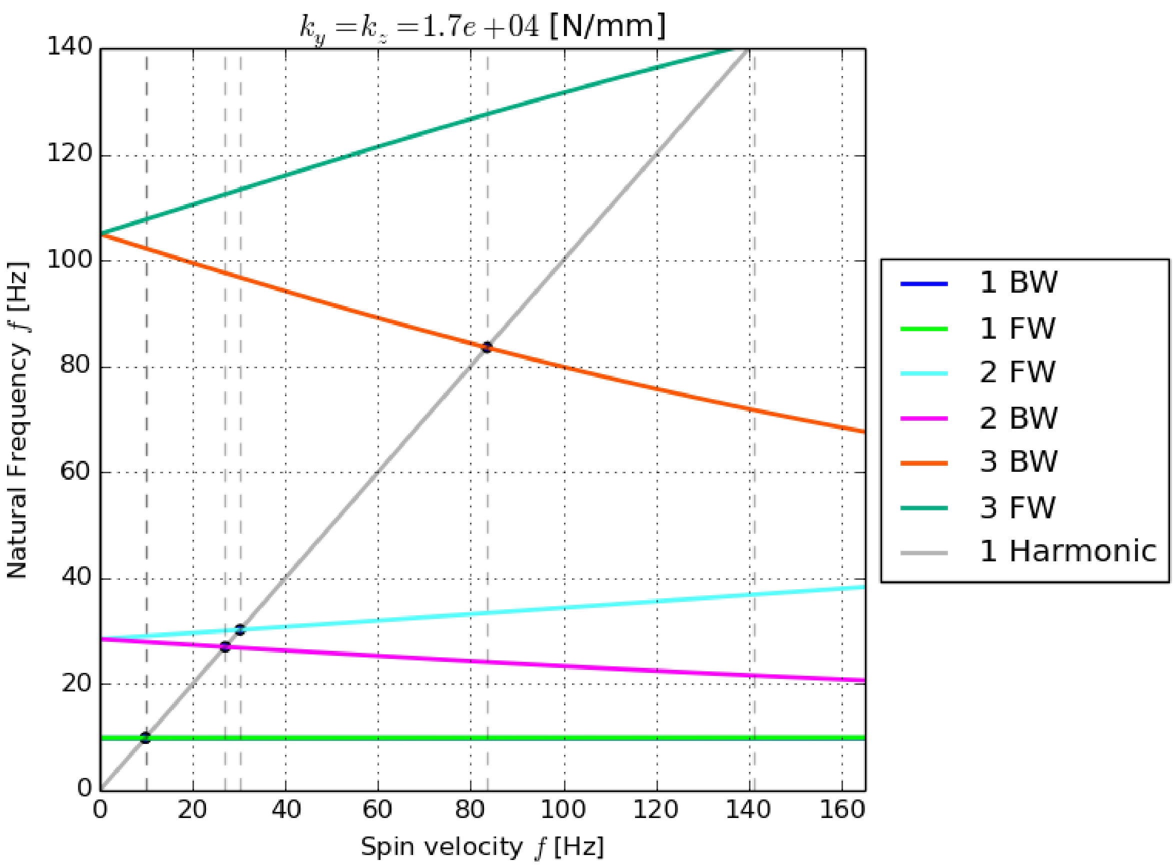

Fig. 1
Campbell diagram

3 Critical Speed Map

Nowadays critical speed maps are used in the early design stage of rotors [7, 8].
Note that the first two modes with low support stiffness involve a negligible amount of shaft bending
Fig. (2) illustrates the natural frequency variation according to bearing stiffness. The first three critical speeds typically vary with support stiffness [1]. In this case, the sensitivity of all critical speeds to support stiffness does not permit a range of operating speeds that does not traverse any of the the critical speeds. A relative strong variation of the critical speeds can be achieved by an adaptation of the support stiffness in the interval $[10^4; 10^7]$ [N/mm].

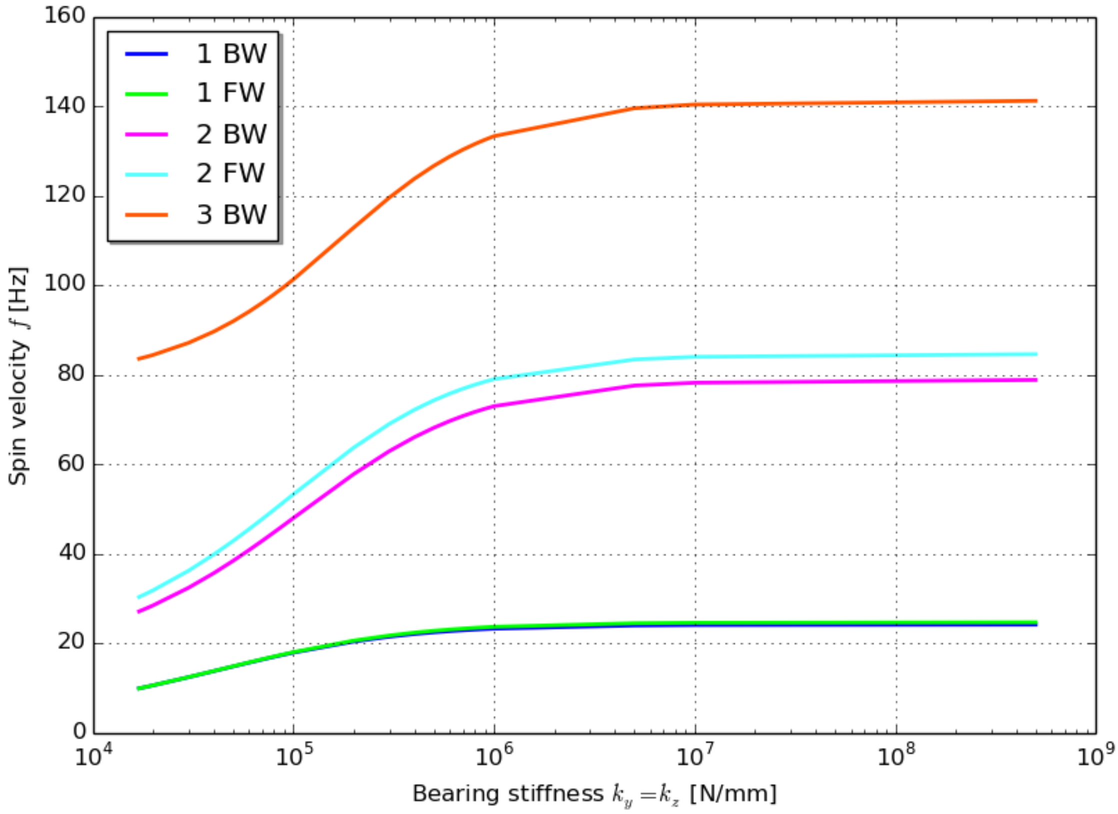

Fig. 2
Critical speed map for accompanying example [6]

4 Conclusions

This paper introduces a two-disk, two-bearing elastic rotor and analyzes its rotordynamics by 3-D finite element analyses. A critical speed map is computed in one computational run using the new SAMPLING procedure of PERMAS and an additional Python script that is directly invoked by PERMAS by using a so-called USER section inside the user control interface.

5 References

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